Friday, August 26, 2015

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Problem 1

Problem. Identify u and du for the integral $\displaystyle\int (8x^2+1)^2(6x)\,dx$.

Solution. Let $u = 8x^2+1$. Then $du = 16x\,dx$.

Problem 3

Problem. Identify u and du for the integral $\displaystyle\int \tan^2 x \sec^2 x\,dx$.

Solution. Let $u = \tan x$. Then $du = \sec^2 x\,dx$.

Problem 9

Problem. Find the indefinite integral $\displaystyle\int x^3(x^4+3)^2\,dx$.

Solution. Let $u = x^4+3$. Then $du = 4x^3\,dx$. Now substitute and integrate.

$$\begin{aligned}\int x^3(x^4+3)^2\,dx &= \frac{1}{4}\int 4x^3(x^4+3)^2\,dx\\ &= \frac{1}{4}\int u^2\,du\\ &= \frac{1}{4}\cdot\frac{1}{3}u^3 + C\\ &= \frac{1}{12}(x^4+3)^3 + C.\end{aligned}$$

Problem 13

Problem. Find the indefinite integral $\displaystyle\int t\sqrt{t^2+2}\,dt$.

Solution. Let $u = t^2+2$. Then $du = 2t\,dt$. Now substitute and integrate.

$$\begin{aligned}\int t\sqrt{t^2+2}\,dt &= \frac{1}{2}\int 2t\sqrt{t^2+2}\,dt\\ &= \frac{1}{2}\int u\,du\\ &= \frac{1}{2}\cdot\frac{1}{2}u^2 + C\\ &= \frac{1}{4}(t^2+2)^3 + C.\end{aligned}$$

Problem 17

Problem. Find the indefinite integral $\int \frac{x}{(1-x^2)^3}\,dx$.

Solution. Let $u = 1 - x^2$. Then $du = -2x\,dx$. Now substitute and integrate.

$$\begin{aligned}
\int \frac{x}{(1-x^2)^3}\,dx &= -\frac{1}{2}\int \frac{-2x}{(1-x^2)^3}\,dx \\
&= -\frac{1}{2}\int \frac{1}{u^3}\,du \\
&= -\frac{1}{2}\int u^{-3}\,du \\
&= -\frac{1}{2}\cdot\frac{1}{-2}u^{-2} + C \\
&= \frac{1}{4}(1-x^2)^{-2} + C. \\
&= \frac{1}{4(1-x^2)^2} + C.
\end{aligned}$$

Problem 21

Problem. Find the indefinite integral $\int \frac{x}{\sqrt{1-x^2}}\,dx$.

Solution. Let $u = 1 - x^2$. Then $du = -2x\,dx$. Now substitute and integrate.

$$\begin{aligned}
\int \frac{x}{\sqrt{1-x^2}}\,dx &= -\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx \\
&= -\frac{1}{2}\int \frac{1}{\sqrt{u}}\,du \\
&= -\frac{1}{2}\int u^{-1/2}\,du \\
&= -\frac{1}{2}\cdot 2u^{1/2} + C \\
&= -\sqrt{u} + C. \\
&= -\sqrt{1-x^2} + C.
\end{aligned}$$

Problem 35

Problem. Find the indefinite integral $\int \cos 8x\,dx$.

Solution. Let $u = 8x$. Then $du = 8\ dx$. Now substitute and integrate.

$$\begin{aligned}\int \cos 8x\ dx &= \frac{1}{8}\int 8\cos 8x\ dx \\ &= \frac{1}{8}\int \cos u\ du \\ &= \frac{1}{8}\sin u + C \\ &= \frac{1}{8}\sin 8x + C.\end{aligned}$$

Problem 39

Problem. Find the indefinite integral $\int \sin 2x \cos 2x\ dx$.

Solution. You may want to work this problem in two stages, using a substitution for each stage. We will use u first, and then v.

Let $u = 2x$. Then $du = 2\ dx$. Now substitute, but do not integrate yet.

$$\begin{aligned}\int \sin 2x \cos 2x\ dx &= \frac{1}{2}\int 2\sin 2x \cos 2x\ dx \\ &= \frac{1}{2}\int \sin u \cos u\ du.\end{aligned}$$

Now let $v = \sin u$. Then $dv = \cos u\ du$. Now we can substitute and integrate.

$$\begin{aligned}\frac{1}{2}\int \sin u \cos u\ du &= \frac{1}{2}\int v\ dv \\ &= \frac{1}{4}v^2 + C \\ &= \frac{1}{4}\sin^2 u + C \\ &= \frac{1}{4}\sin^2 2x + C.\end{aligned}$$

Problem 55

Problem. Evaluate the definite integral $\int_{-1}^{1} x(x^2+1)^3\ dx$.

Solution. Let $u = x^2+1$. Then $du = 2x\ dx$. Now substitute and integrate. Note that $u(-1) = 2$ and $u(1) = 2$.

$$\begin{aligned}
\int_{-1}^{1} x(x^2+1)^3\,dx &= \frac{1}{2}\int_{-1}^{1} 2x(x^2+1)^3\,dx \\
&= \frac{1}{2}\int_{-1}^{1} 2x(x^2+1)^3\,dx \\
&= \frac{1}{2}\int_{u(-1)}^{u(1)} u^3\,du \\
&= \frac{1}{8}\left[u^4\right]_2^2 \\
&= 0.
\end{aligned}$$

Problem 57

Problem. Evaluate the definite integral $\int_1^2 2x^2\sqrt{x^3+1}\,dx$.

Solution. Let $u = x^3 + 1$. Then $du = 3x^2\,dx$. Now substitute and integrate. Note that $u(1) = 2$ and $u(2) = 9$.

$$\begin{aligned}
\int_1^2 2x^2\sqrt{x^3+1}\,dx &= \frac{2}{3}\int_1^2 3x^2\sqrt{x^3+1}\,dx \\
&= \frac{2}{3}\int_2^9 \sqrt{u}\,du \\
&= \frac{2}{3}\left[\frac{2}{3}u^{3/2}\right]_2^9 \\
&= \frac{4}{9}\left(9^{3/2} - 2^{3/2}\right) \\
&= \frac{4}{9}(27 - 2\sqrt{2}) \\
&= 12 - \frac{8\sqrt{2}}{9}.
\end{aligned}$$

Problem 61

Problem. Evaluate the definite integral $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$.

Solution. Let $u = 1 + \sqrt{x}$. Then $du = \frac{1}{2\sqrt{x}}\,dx$. Now substitute and integrate. Note that $u(1) = 2$ and $u(9) = 4$.

$$\begin{aligned}
\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx &= 2\int_1^9 \frac{1}{2\sqrt{x}(1+\sqrt{x})^2}\,dx \\
&= 2\int_2^4 \frac{1}{u^2}\,du \\
&= 2\left[-\frac{1}{u}\right]_2^4 \\
&= 2\left(-\frac{1}{4}+\frac{1}{2}\right) \\
&= \frac{1}{2}.
\end{aligned}$$

Problem 65

Problem. Find the area of the region represented by $\displaystyle\int_0^7 x\sqrt[3]{x+1}\,dx$.

Solution. Let $u = x + 1$. Then $du = dx$. Now substitute and integrate. Note that $u(0) = 1$ and $u(7) = 8$. Also, note that $x = u - 1$.

$$\begin{aligned}
\int_0^7 x\sqrt[3]{x+1}\,dx &= \int_1^8 (u-1)\sqrt[3]{u}\,du \\
&= \int_1^8 (u-1)u^{1/3}\,du \\
&= \int_1^8 (u^{4/3}-u^{1/3})\,du \\
&= \left[\frac{3}{7}u^{7/3}-\frac{3}{4}u^{4/3}\right]_1^8 \\
&= \frac{3}{7}\left(8^{7/3}\right)-\frac{3}{4}\left(8^{4/3}\right)-\left(\frac{3}{7}-\frac{3}{4}\right) \\
&= \frac{3}{7}(128)-\frac{3}{4}(16)-\left(\frac{3}{7}-\frac{3}{4}\right) \\
&= \frac{3}{7}(127)-\frac{3}{4}(15) \\
&= \frac{381}{7}-\frac{45}{4} \\
&= \frac{1209}{28}.
\end{aligned}$$